Lec 6: 09/10/2018

Particle Acceleration (Cont'd):

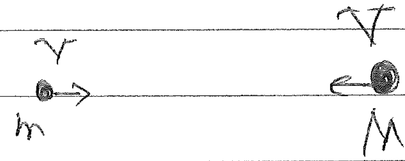
Fermi Acceleration:

Gas flowing into a compact object is often supersonic. This means that sound waves form discontinuous jumps across the resultant shock front. The diffusion of particles across the shock accelerates them and produces a power-law distribution, which is called the "Fermi acceleration". The accelerating particles scatter off the random fluctuations in the magnetic field (which is a collisionless process). This mechanism constitutes the second electromagnetic acceleration scheme.

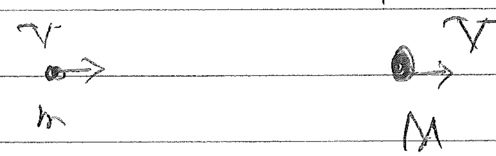
To see how random scattering of particles between molecular clouds in the interstellar medium can accelerate them, lets consider a simple one-dimensional version of the problem.

We consider a large number of massive particles with mass M (the clouds) moving with the same speed V but randomly (i.e., back and forth). They collide with a small test particle of mass m, whose velocity is v. There are two types of collisions in this set up:

(1) Head-on Collision (h):



(2) Catch-up Collision (c):



Here we assume that $m \ll M$ and $v \gg V$.

By using the conservation of momentum and energy, one can show that the energy of the test particle changes by respective amounts ΔE_h and ΔE_c in head-on and catch-up collisions, where:

$$\Delta E_h = \frac{1}{2} m (v+2V)^2 - \frac{1}{2} m v^2 = 2m v V + 2m V^2$$

$$\Delta E_c = \frac{1}{2} m (v-2V)^2 - \frac{1}{2} m v^2 = -2m v V + 2m V^2$$

It is seen that the test particle gains energy in head-on collisions and loses energy in catch-up collisions. To find the net increase in the energy, we should also include the probability for each type of collision. The probability is proportional to the collision rate, which is in turn proportional to the relative velocity of colliding particles. We have:

$$P_h = \frac{v+V}{2v}, \quad P_c = \frac{v-V}{2v}$$

Thus, the average increase in energy per collision is:

$$\langle \Delta E \rangle = P_h \Delta E_h + P_c \Delta E_c \Rightarrow \frac{\Delta E}{E} = 4\left(\frac{V}{v}\right)^2$$

This is a second-order effect, which is too small to yield a significant energy increase. However, as we will see, a shock geometry improves the situation by producing first-order dependence of ΔE on the velocities.

Before turning to this, we can also find the energy spectrum resulting from this stochastic process by using the one-dimensional model. The energy distribution of particles satisfies the diffusion loss equation:

$$\frac{dN(E,x,t)}{dt} = -\frac{\partial \phi_s}{\partial x} - \frac{\partial \phi_E}{\partial E} + \frac{\partial N}{\partial t}$$

Here $N(E,x,t)$ is the distribution function, and $N\,dE\,dx$ is the number of particles at time t between x and $x+dx$ whose energy is between E and $E+dE$. ϕ_s is the spatial flux, which can be written in terms of a diffusion coefficient D:

$$\phi_s = -D\frac{\partial N}{\partial x}$$

ϕ_E is the energy flux, and is given by:

$$\phi_E = N\frac{dE}{dt}$$

Here $\frac{dE}{dt}$ is the rate of energy increase due to acceleration of particles. It follows;

$$\frac{dE}{dt} = \langle \Delta E \rangle \nu = 4\nu\left(\frac{V}{v}\right)^2 E$$ (ν: frequency of collisions)

The term $\frac{\partial N}{\partial t}$ represents the explicit time variation in N. We can simplify the diffusion-loss equation by ignoring diffusion and setting $\frac{\partial N}{\partial t} = \frac{-N}{\tau}$, where τ is the characteristic time scale that particles spend in the acceleration zone. We then find:

$$\frac{dN}{dt} \simeq -\frac{\partial}{\partial E}(N\alpha E) - \frac{N}{\tau} \qquad \alpha \equiv 4\nu \left(\frac{V}{v}\right)^2$$

In equilibrium, we have $\frac{dN}{dt} = 0$, which gives rise to:

$$\frac{\partial (N\alpha E)}{\partial E} \simeq -\frac{N}{\tau} \Rightarrow \alpha N + \alpha E \frac{\partial N}{\partial E} \simeq -\frac{N}{\tau} \Rightarrow \frac{\partial N}{\partial E} \simeq -\left(1+\frac{1}{\alpha\tau}\right)\frac{N}{E}$$

If $\alpha\tau$ is constant, we find:

$$N(E) \simeq N_0 E^{-\left(1+\frac{1}{\alpha\tau}\right)}$$

This is a remarkable result that demonstrates the energy spectrum is a power-law function, as seen in the case of cosmic rays and many high-energy sources. The spectral index $-\frac{1}{1+\alpha\tau}$ may vary slightly from location to location.